The work of Fernando de Helguero on non-normality arising from selection

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Skew-symmetric/hidden-truncation models

- much recent work about distributions like
$$f(z) = \text{const} \times f_0(z)\, G(\lambda_0 + \lambda_1\, z)$$
- strongly linked to a selection mechanism
$$Z = (Z_0 | X < \lambda_0 + \lambda_1\, Z_0)$$
- main special case
$$f(z) = \text{const} \times \varphi(z)\, \Phi(\lambda_0 + \lambda_1\, z)$$
- this mechanism features marginally in several older papers
- . . . but obtained as a by-product in different type of problems
- Q: when did this construction appear first with the aim of *building an alternative to the Gaussian family*?

Fernando de Helguero (1880-1908)

Fernando de Helguero, in brief

- born in 1880 near Florence, Italy
- studied mathematics at Univ. Rome
- teacher of mathematics while student of natural sciences
- mathematics + biology $\Rightarrow$ statistics and biometry
- smart, active, excellent publications, but...
- deceased at age 28 in earthquake

Here we focus on a specific and nearly unknown of his contributions

Historical background

Distribution theory at in the beginning of 20th century

Gaussian distribution was the 'normal' one,
but wider families were called for

- Gram-Charlier and Edgeworth expansions
- mixture of normals, Pearson's work
- Pearson system of 12 families

The study of data distribution was a key target of statistics

de Helguero's view

FdH views on non-Gaussian data distributions

- principle: the role of statistics is not simply to fit data, but also to help understanding the underlying mechanism
- $\Rightarrow$ Edgeworth expansions and Pearson system ☹
- mixtures (of normals) are more helpful
- in 1908, at 4th ICM, FdH presents an innovative proposal

de Helguero's formulation

- A real phenomenon would generate a Gaussian distribution
$$X \sim \mathrm{N}(b, \sigma^2)$$
if some external 'cause' did not perturb it
- often a realistic perturbation mechanism is that a value x is censored with probability $\varphi(x)$
$\Rightarrow$ a curve "perturbed by selection"
- hence the observable distribution is proportional to
$$\mathrm{const} \times \frac{1}{\sqrt{2\,\pi\sigma}} \exp\left\{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2\right\}[1-\varphi(x)]$$
- $\varphi(x)$ can take various forms, the simplest is linear:
$$\varphi(x) = A(x-b) + B$$

Modern reading

$$\text{const} \times \frac{1}{\sqrt{2\,\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2\right\} \underbrace{[1 - B - A(x-b)]}_{1-\varphi(x)}$$

- $1 - \varphi(x)$ is linear in the interval of main interest, constrained to be 0 to the left, 1 to the right (or vice versa)
- in current view, $\mathrm{N}(b, \sigma^2)$ with a uniform 'skewing factor'
- in this sense FdH anticipates the key ingredients of the current 'skew' literature

de Helguero's formulation (ctd)

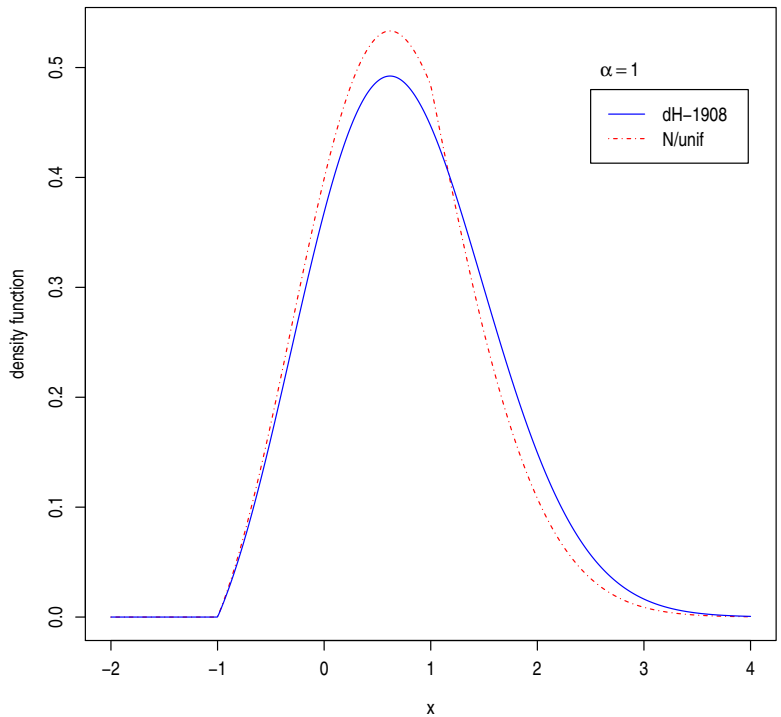
A further step:

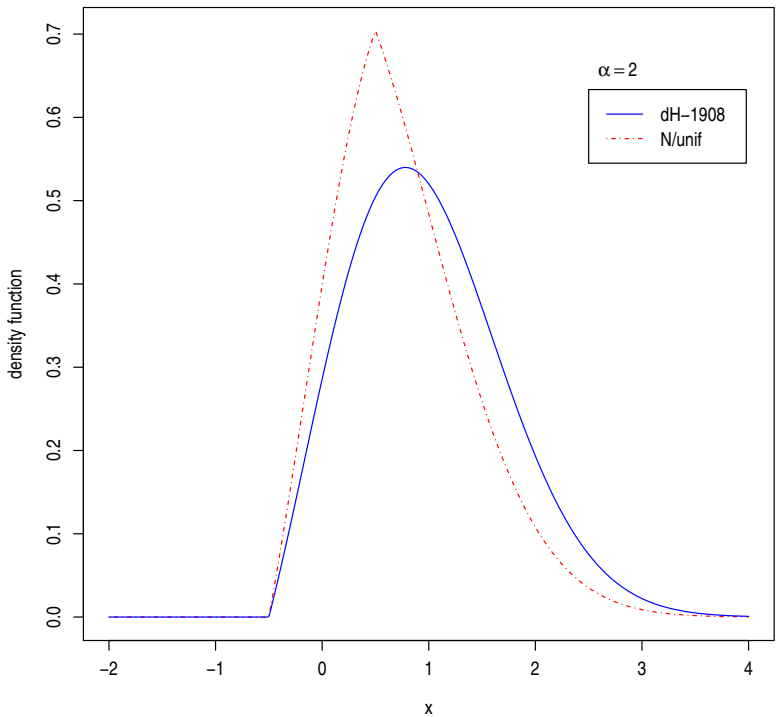
- in addition to *thinning* by censoring,
  FdH includes the possibility of *thickening* a tail
- the same mathematical expression holds
- only the 0 constraint is retained for $1 - \varphi(x)$, it can exceed 1

Operative stage:

- FdH obtains expressions of moments up to order 3
- data fitting by method of moments
- develop special tables to solve non-linear equations
- various numerical illustrations with real data

Some de Helguero's distributions





Final comments

- FdH anticipated the key idea of current 'skew distributions'
- because of his premature passing his curves "perturbed by selection" went unnoticed
- if he had survived, he would have been a major role in the Italian statistical and biometrical world, not only for this specific theme.

References

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